PERSONAL EQUATION.[1]

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II.

VARIATIONS IN THE AMOUNT OF PERSONAL EQUATION.

A certain variability in the personal equation is generally admitted. Those that assert its constancy, like Wolf,[2] separate it into two factors, one depending on the circumstances of observation, the other on the observer, and mean that the last is practically constant. But even this must not be taken too strictly. There are changes of personal equation while the circumstances of observation so far as known remain the same—progressive for some observers; quite without order for others. Both are illustrated in the equations of Main, Rogerson and Henry at Greenwich from 1841 to 1853 (observations by eye and ear). From 1846

[1] Continued from AMERICAN JOURNAL OF PSYCHOLOGY, November, 1888. The last section of the first part dealt with the absolute personal equation, and the last paragraph but one with a method of getting the absolute personal equation from transits of real stars. The following wholly different method, applied by Bredichin (*Annales de l'Observatoire de Moscou*, Vol. II, part 2, page 69), may be mentioned here. He replaced the central wire by a plate of metal having its sides parallel to the other wires. He observed at the wires in the usual way and by immergences and emergences at the edges of the plate. The mean of the last two gives approximately the true time of the transit of the middle of the plate (within 0.02 s for Bredichin), from which the personal equation for the wires can be obtained.

[2] Wolf: *Annales de l'Obs. de Paris, Mémoires* VIII, 153. Hough: American Assoc. Proceedings, XVI, 22. Wolf admits changes through fatigue, etc.

the calculations are given by the Astronomer Royal,[1] the earlier ones are supplied by Peters.[2]

	M—R	M—H
	s.	s.
1840	= −0.15	= ..
1841	= +0.08	= −0.09
1842	= ..	= −0.01
1843	= +0.20	= −0.02
1844	= +0.18	= −0.05
1845	= +0.20	= −0.12
1846	= +0.26	= −0.05
1847	= +0.35	= −0.03
1848	= +0.37	= −0.04
1849	= +0.39	= −0.05
1850	= +0.45	= −0.11
1851	= +0.47	= −0.11
1852	= +0.63	= −0.00
1853	= +0.70	= +0.03

The increase of M—R is said by Dunkin, a fellow observer, to be due to a change in Rogerson's method of observing, but he fails to specify the nature of the change.

Further evidence of variability is furnished by changes in the difference between the personal equation for eye and ear and chronographic observations. Hilfiker found differences in his own personal equation by the two methods as follows:[3] (1883) 0.074 s, (1884) 0.086 s, (1885) 0.053 s, (1887) 0.022 s. Those below are given for the Greenwich observers:[4]

EXCESS OF CLOCK CORRECTION BY THE EYE AND EAR METHOD.

	HT.	AD.	T.	L.	H.
	s.	s.	s.	s.	s.
1885	−0.02	+0.06	+0.04	+0.10	+0.63
1886	+0.05	+0.08	+0.05	+0.12	+0.59
1887	+0.03	+0.05	+0.02	+0.12	+0.58

[1] Greenwich Observations.

[2] *Astronomische Nachrichten*, XLIX, 1.

[3] *Astronomische Nachrichten*, No. 2815.

[4] *Observatory*, Jan. 1888, p. 95.

The personal equation varies even from day to day. To mention only two from many instances, I may refer to the comparisons before mentioned, of Nehus and Wolfers, and those of Hirsch and Plantamour.

The personal equation for the sun and moon is different from that for the stars; and that for the first limb of the sun and moon is different from that for the second.—The observation of star transits by both methods has been described. In observing transits of the sun and moon, it is the custom not to bisect the disk, but to observe the transits of the limbs, and from them to get the time for the center. Astronomers have generally had a different feeling in observing stars and limbs; it is one thing to estimate the distance of the edge of a large bright disk from the wire or to fix the contact, and quite another to do the same thing for a star. It is also more difficult to observe the advancing edge when the wires are faint from the glare, than the retreating edge when the wires are black lines on a bright surface.

The feeling of the astronomers is justified by the probable errors. Dr. Robinson,[1] observing at Armagh in 1830 by the eye and ear method, found for a transit of the sun's first limb a probable error of ± 0.116 s, for the second ± 0.087 s, for stars ± 0.097 s. Downing found[2] from a hundred complete transits of the sun at Greenwich in 1874 and 1875, as follows:

	I Limb.	II Limb.	Star.
	s.	s.	s.
Prob. error of transit of one wire,	± 0.072	± 0.063	± 0.051
Prob. error of complete transit,	± 0.024	± 0.021	± 0.017

[1] Proceedings of the Royal Irish Academy, VII, 371.

[2] *Monthly Notices*, XXXVIII, 102.

Other errors are here involved, but not such as to affect the relative amounts.

Besides the difference in certainty, the presence of a real difference of personal equation for the two limbs was early recognized. In 1848 a large discordance was found in the tabular errors of the moon in right ascension as deduced from the altazimuth observations of Dunkin and those of H. Breen. From a year's observations of the first limb the difference amounted to 0.46 s, from those of the second to 0.30 s. In 1869 Dunkin investigated the subject on a basis of six years' observations of the moon with the altazimuth and transit circle, and in 1874, on a basis of ten years' transit observations of the sun.[1] Five years later Neison studied the transit observations of the moon from 1863 to 1876.[2] The results are all in substantial agreement, as shown by the following tables:

Observations on the Moon.

DUNKIN, 1869:

	Transit circle.			Altazimuth.	
	I Limb.	II Limb.		I Limb.	II Limb.
C—D	+0.034	+0.032	C—D	—0.047	—0.028
C—E	+0.112	+0.077	C—E	+0.104	+0.048
C—JC	+0.132	+0.038	C—JC	+0.207	+0.004

NEISON, 1879:

RELATIVE PERSONAL EQUATIONS.[3]

	I Limb.	Wt.	II Limb.	Wt.
C—D	=+0.012	4	+0.032	3
C—E	=+0.106	9	+0.065	7
C—JC	=+0.125	8	+0.038	6
C—L	=—0.034	4	—0.058	3
C—AD	=—0.007	2	—0.109	2
C—T	=—0.040	1	—0.093	½

[1] *Monthly Notices*, XXIX, 259; XXXV, 91.

[2] *Monthly Notices*, XL, 75.

[3] The signs in this table are changed, to make comparison with the rest more easy.

Absolute Personal Equations.

	I Limb.	II Limb.
	s. s.	s. s.
D	+ 0.025 ± 0.012	+ 0.004 ± 0.014
E	— 0.060 ± 0.009	— 0.041 ± 0.010
C	+ 0.045 ± 0.008	+ 0.026 ± 0.009
JC	— 0.074 ± 0.010	— 0.014 ± 0.012
L	+ 0.075 ± 0.015	+ 0.066 ± 0.020
AD	+ 0.006 ± 0.022	+ 0.132 ± 0.026
T	+ 0.063 ± 0.025	+ 0.183 ± 0.050

The numbers in the first table are really mutual differences, obtained from averages of the tabular errors of the moon as found by the different observers, and contain, especially those for the altazimuth, other errors than those of personal equation. Those in the second table are averages of the mean personal equations of the fourteen years, weighted for number and distribution of observations. Those of the third are calculated on the assumption that the personal equations compensate each other and that their sum equals zero. The results for A, D, and T are less certain, because of the small number of their observations.

Observations on the Sun.[1]

Dunkin, 1874:

	I Limb.	II Limb.
C—D	+ 0.042	+ 0.002
C—E	+ 0.103	+ 0.019
C—JC	+ 0.150	— 0.001
C—L	+ 0.016	— 0.037
C—HC	+ 0.034	— 0.126

The differences come out most strongly for most observers in transits of the first limb where the glare is most troublesome, but not for all. Dunkin suggests no explanation but a difference in the "methods of estimating by the eye the positions of the two limbs in

[1] The last two personal equations are not in Dunkin's table, but are figured from other tables given.

transit as affected by irradiation." Dr. Robinson also assigned irradiation as the cause, and by lessening it was able to lessen the error materially.

The personal equation changes with the magnitude of the star.—An early notice of such a connection, perhaps the earliest, comes from the elder Struve.[2] About 1869, the attention of Argelander was called to a difference of 0.66 s in the right ascension of the minor planet Egeria as determined at Bonn and Leyden. An examination of the observations, and exclusion of other sources of error, reduced the possible causes to a different perception of the instant of transit for faint and bright stars. He had before suspected that he himself observed the transits of very faint stars differently from those of bright ones. To test his suspicion, he examined his observations of variable stars as far as the ninth magnitude, but without finding any appreciable difference. With still fainter stars, 9.2 and 9.3 magnitudes, he found a slight tendency to observe too early, but not more for the latter than 0.15 s. Since this was insufficient to account for the difference in the position of Egeria, he suggested a variation of the same kind in the observatory of Leyden, where the observer might have been less experienced.

It was, indeed, at that observatory, though several years later, that the question was first thoroughly investigated.[3] In connection with a study of the orbit

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[1] Introduction to Armagh Catalogue.

[2] *Positiones Mediae*, lxiv.

[3] In the same year as Argelander's demonstration of the difference, the question was touched experimentally by Rogers (*American Journal of Science and Arts*, Second series, Vol. XLVII, p. 297). His experiments dealt with several other causes of variation than the size of the star, and though, in comparison with later experiments, his are crude and, as the author himself recognizes, not

of Mars by Gill, in 1877, the place of a number of stars was fixed at the same time by twelve important observatories in different parts of the world. The resulting right ascensions showed differences that Gill traced, in part at least, to variations of personal equation with magnitude. Following such a suggestion, previously made to him by Gill, H. G. v. d. Sande Bakhuyzen, of the observatory of Leyden, reviewed the results. On the assumption that the mean of the right ascensions from the ten observatories (two were omitted for special reasons) was exact, he found the differences of each from the mean and plotted curves, to show the effect in each of a change of brightness in the star.[1] The form of these curves is distinct for the different observatories, and very interesting, but on a narrow numerical basis.

To put the question to a direct experimental test, Bakhuyzen and his assistants observed a number of transits, using for half of each the full objective, and,

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definitive, they will be referred to several times in what follows—a word, therefore, as to the manner of them. The artificial stars were made of paper and mounted on wires attached to a chronograph drum. Their transits behind a fixed wire and the record of the observer were taken electrically on the same drum. This record was measured to tenths and estimated to hundredths of a second. In general, eight stars were used, and ten turns of the drum made a series. In his experiments on magnitude, he and his assistant observed the transits of five paper stars, the first of which was larger than the rest. The result, in the form of relative personal equations, was: For the large star, R—T = —0.233 s; for other stars, R—T = —0.152 s; difference, 0.081 s. To show the same result in another way: the positive excess of T's personal equation for the large star over the small ones was 0.047 s; B's negative excess was 0.036 s; the difference of R and T, 0.083 s. No such clear difference could be traced in the observation of real stars. The experiments evidently do not touch the question of brightness apart from apparent area of the disk.

[1] These curves accompany Gill's article, *Monthly Notices*, XXXIX (1879), 98, which is of later date than Bakhuyzen's communication in the *Astronomische Nachrichten*, XCV, 187. Gill touches the question also in the same volume of the *Monthly Notices*, p. 434.

for the other half, the objective reduced by a diaphragm to $\frac{1}{7.37}$. This reduced the stars about 2.3 magnitudes. To avoid errors, the diaphragm was used alternately for the first and second halves. These are the mean results for observations by the chronograph method :

REDUCED MINUS FULL OBJECTIVE.

	First half transit reduced.	No. of stars.	Second half transit reduced.	No. of stars.
E. F. Bakhuyzen,	+0.071	8	+0.070	8
Wilterdink,	+0.049	11	+0.047	8
Stieltjes,	+0.060	8	+0.034	9

Continuing his experiments, Bakhuyzen later published experiments with both methods of observing, in part of which the diaphragm was used, and in part a grating of fine copper wire that reduced the stars 2.8 magnitudes.[1] The following are the mean results :[2]

REDUCED MINUS FULL OBJECTIVE.

Chronograph Method.

	s. s.	Mean mag.	No. of obs.
E. F. Bakhuyzen,	0.049 ± 0.007	5.5	80
	0.045 ± 0.013	3.1	14
Wilterdink,	0.039 ± 0.009	5.3	48
	0.035 ± 0.019	2.5	5
Stieltjes,	0.041 ± 0.008	5.3	65
	0.045 ± 0.012	3.1	15

Eye and Ear Method.

E. F. Bakhuyzen,	0.054 ± 0.013	5.1	41
	0.065 ± 0.023	3.0	8
Wilterdink,	0.023 ± 0.020	5.4	16
	0.128 ± 0.033	2.1	4
Stieltjes,	0.041 ± 0.014	5.5	37
	0.103 ± 0.037	3.5	3

These tables show, in brief, that all the observers at Leyden observed bright stars too early by both methods.

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[1] *Vierteljahrssch. d. Astronom. Gesell.* XIV, 408.

[2] The mean magnitudes in the table are for full objective.

This increase in personal equation as magnitude diminishes probably has more than one cause. Wundt sees in it a close parallel of the increase in the length of the simple reaction time when the stimulus is diminished.[1] That is due to a change from the motor form of reacting, in which the attention is centered on executing the motion of recording, to the sensory form, where it is centered on catching the phenomenon to be recorded. Indeed, he suggests this kind of change as a general explanation of the change of relative personal equation. A change of attention might also affect observations with the eye and ear, as well as those with the chronograph.[2] In some cases this further cause also co-operates. It was explained in the first part of this paper that some observers, in using the chronograph method, anticipate the actual transit. They tap the key so as to make the sound or the muscular sensation of the closing coincide with the bisection.[3] If an observer does so, "he will be nearly certain," says Gill, "to press too soon for very bright, and too late for very faint stars, because the larger disk and the rings of a bright star will make it appear closer to the web than the small, sharp disk of a faint star, at the same angular distance from the web."

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[1] *Physiologische Psychologie*, 3d ed., II, 273.

[2] Compare the observations of Hartmann, mentioned in the first part of this paper, p. 29.

[3] This way of observing, if not the best astronomically, is no doubt the natural one—a special case of the general tendency that makes man forethoughtful. All the persons tested by Mitchel (first part, p. 27) anticipated. That it is common among astronomers also, may be argued from the size of the personal equations. The shortest reaction-times found by psycho-physical experimenters for recording the reception of an instantaneous and expected stimulus (a sound or a certain position of a pointer on a dial) are about 0.075 s. It is clear, therefore, that observers whose records follow the actual bisection by less than about 0.075 s begin the motion of recording before the phenomenon has actually appeared. Personal equations by the chronograph method of less than 0.075 s are, I believe, by no means exceptional.

Yet another cause is suggested by Newcomb.[1] In comparing the right ascensions given by observations on the transit circle and transit, he found that those given by the former for faint stars were too great. After explaining that the spider lines appeared finer and the field less brightly illuminated in that instrument, he continues: "The effect of the latter cause will probably be to make an observer later in recording a transit. . . . The stray light which surrounds a bright star forms a bright ground for the dark transit wire a perceptible time before the star reaches the wire, and thus the approach of the two objects is distinctly seen. As we take fainter stars the stray light disappears and the approach is less distinctly seen. Thus the effect in question will be exaggerated as the star grows fainter."

The personal equation is influenced by the direction of the star's motion.—I have already mentioned, in summarizing Wolf's experiments (first part, page 36), that his personal equation was 0.04 s greater when the star moved from left to right than from right to left. In a single experiment, where he lay down at right angles to his telescope and thus observed with the star moving along the vertical meridian of his eye, his personal equation was 0.127 s, against 0.118 in the ordinary position. Kaiser found (first part, page 32) only one of the four observers at Leyden who was affected by change of horizontal direction, though all four were affected by change from a horizontal to a vertical direction. About the same time (1867), in a treatise presented to the Austrian Academy of Sciences, K. von Littrow discussed the differences of

[1] Observations at the United States Naval Observatory, XIV (1867), Appendix III, p. 27.

personal equation in the east and west positions of the "broken transit" or reflection theodolite.[1]

In this instrument, to facilitate observation and give greater steadiness, the rays of light gathered by its objective are bent at right angles by a prism at the level of its transit axis, where one of the trunnions is made to do duty for the ocular end of the tube. In an ordinary transit instrument, stars south of the zenith appear to move from right to left, between the zenith and the pole from left to right, and below the pole again from right to left. Zenith stars themselves may be observed with motion in either direction according to which side of the instrument the observer places himself. In the "broken transit," however, the apparent motions are vertical or oblique. Suppose such an instrument set so that the telescope tube moves in the plane of the meridian, and the trunnion eye-piece points east. The observer then faces west, and without changing his position can sweep the whole of his meridian from the northern to the southern horizon. He can do the same thing when the instrument is turned around and he faces east. But there is this difference: when the observer faces west, all stars south of the pole seem to move upward in his field of view; zenith stars directly upward, stars between the zenith and the pole upward from left to right, those south of the zenith upward from right to left, and stars below the pole obliquely downward from right to left. When the observer faces east the direction of motion is reversed: all stars south of the pole seem to move downward, those between the zenith and the pole obliquely from left to right, south of the zenith obliquely from right to left, and below the pole

[1] *Astronomische Nachrichten*, LXVIII, 369.

obliquely upward from right to left. Observations on such an instrument are, therefore, admirably suited to bring out any differences that depend on direction of motion.

Differences of clock corrections found in the two positions of the instrument were noticed in 1864 by Weiss; and in 1865, differences between the personal equations of Föster and Weiss in the two positions were also noticed. The latter then calculated from his earlier figures, making certain assumptions, the difference of personal equation in the two positions. The amounts are somewhat irregular, but the following are the means found:[1]

Observer East Minus Observer West.

	Weiss.	Bruhns.
	s.	s.
Eye and ear,	—0.166	+0.072
Chronograph,	—0.214	—0.099

The observations on which these figures rest were of stars at about the same distance from the pole, and, therefore, cannot show the changes that very likely enter when that distance is changed, and with it the rate and obliquity of motion.

Littrow found further proof of the reality of the difference introduced by the direction of motion, in the tests made at Greenwich in 1852 and 1853 with the "binocular eye-piece" (first part, page 22), but never

[1] Bredichin found by his method for absolute personal equation from real stars the following values for his personal equation in the two positions: observer east, first series —0.004, second series —0.015; observer west, first series 0.195, second series 0.241. The observations were by eye and ear on equatorial stars.

Hilfiker found the difference between his personal equation by the two methods to be different in the two positions of the instrument: observer east 0.024 s, observer west 0.018 s. For experiments incidentally touching direction of motion see also Buccola: *La legge del tempo*, p. 162.

before worked up. To observers comparing for personal equation by means of this eye-piece, the stars would appear to move in the same direction, right and left, as with the ordinary eye-piece ; but to the one at the east branch, stars south of the pole would appear to move obliquely upward, to the observer at the west branch obliquely downward ;[1] for stars below the pole the directions would be reversed. From Littrow's tabulated results (observations by eye and ear only) I take a few for illustration :

D — E (ORDINARY PERSONAL EQUATION — 0.07).

	D east.	Stars.	D west.	Stars.
	s.		s.	
1853—Oct. 14	0.00	3	—0.21	3
20	—0.01	3	—0.17	3
Nov. 9	—0.24	3	—0.56	3

H — D (ORDINARY PERSONAL EQUATION + 0.13).

	H east.		H west.	
	s.		s.	
1852—Feb. 10	+0.27	3	+0.15	3
12	+0.28	2	—0.04	2
Mar. 12	+0.07	3	+0.11	3

R — D (ORDINARY PERSONAL EQUATION + 0.50).

	D east.		D west.	
	s.		s.	
1852—Jan. 23	—0.69	4	—0.52	4
April 2	—0.89	3	—0.47	3
26	—0.65	3	—0.21	3

In 1870 and 1871 Wagner incidentally made determinations with motions toward the right and left in studying his absolute personal equation with a Kaiser machine.[2] His experiments with the two directions were unfortunately not made at the same time and do not show constant differences. The figures, however,

[1] Zenith stars would of course seem to move straight up and down.

[2] *Observations de Poulkova*, Vol. XII.

are given below in the section on the effect of rate (page 288) and may there be consulted.

A few years later Campbell and Heaviside, in connection with longitude work in India, also found differences of personal equation depending on the direction of motion.[1] Their longitude observations were made by the chronographic method, and with an eye-piece that showed the stars in their actual motions —that is, stars north of the zenith with motion from right to left, and stars south with motion from left to right. The relative personal equation for the first was $C - H = 0.124\ s \pm 0.0087\ s$; for the second, $C - H = 0.040\ s \pm 0.0034\ s$, Heaviside observing a little earlier than Campbell. From twenty-one stars near the zenith observed by Campbell through half their transit as north stars and through the other half as south stars (the order being reversed, of course, to exclude error), the following equation results: $N - S = 0.077\ s \pm 0.0067\ s$, motion from left to right being observed a little earlier than that in the contrary direction.

The most recently published and most satisfactory investigation of this question, though confined to motions left and right in a horizontal direction, was also made at the Greenwich Observatory.[2] An artificial transit apparatus on the plan of C. Wolf's, but improved at many points, was set up in 1885, and in 1887 the regular transit observers made special determinations of absolute personal equation with it for the effect of rate and direction. From the report of these by H. Turner I quote the following table. All the observations were by the chronographic method:

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[1] *Monthly Notices*, XXXVII (1877), 283.

[2] *Monthly Notices*, XLVIII, 4.

ABSOLUTE PERSONAL EQUATIONS WITH THE GREENWICH PERSONAL EQUATION MACHINE.

Observer.	Direction.	Rate one half as fast again as the equatorial.	No. of transits.	Equatorial rate.	No. of transits.	Rate at N. P. D. 43½°.	No. of transits.	Rate at N. P. D. 30°.	No. of transits.	Rate at N. P. D. 15°.	No. of transits.
		s. s.		s. s.		s. s.		s. s.		s. s.	
Turner........	l.	-0.053 ± 0.013	6	-0.065 ± 0.015	6	-0.098 ± 0.020	5	-0.115 ± 0.028	4		
	r.	-0.063 ± 0.013	6	-0.043 ± 0.015	7	-0.048 ± 0.020	5	-0.045 ± 0.028	4		
Downing......	l.	$+0.094 \pm 0.007$	18	$+0.064 \pm 0.008$	19	$+0.045 \pm 0.011$	15	$+0.036 \pm 0.014$	15	$+0.052 \pm 0.026$	13
	r.	$+0.072 \pm 0.007$	17	$+0.075 \pm 0.008$	19	$+0.065 \pm 0.011$	15	$+0.076 \pm 0.015$	14	$+0.137 \pm 0.026$	13
Thackeray....	l.	-0.042 ± 0.006	26	-0.077 ± 0.007	25	-0.108 ± 0.009	22	-0.158 ± 0.014	17	-0.109 ± 0.024	15
	r.	-0.044 ± 0.006	26	-0.036 ± 0.007	25	-0.062 ± 0.009	22	-0.069 ± 0.014	18	-0.019 ± 0.024	15
Lewis........	l.	$+0.020 \pm 0.006$	25	$+0.003 \pm 0.007$	22	$+0.003 \pm 0.010$	20	-0.008 ± 0.012	21	-0.077 ± 0.022	18
	r.	$+0.013 \pm 0.006$	24	$+0.021 \pm 0.008$	20	$+0.014 \pm 0.010$	20	$+0.036 \pm 0.013$	20	$+0.093 \pm 0.022$	17
Hollis........	l.	$+0.221 \pm 0.007$	18	$+0.256 \pm 0.007$	23	$+0.260 \pm 0.012$	14	$+0.316 \pm 0.013$	19	$+0.457 \pm 0.023$	16
	r.	$+0.234 \pm 0.007$	19	$+0.267 \pm 0.007$	23	$+0.274 \pm 0.012$	14	$+0.349 \pm 0.013$	19	$+0.528 \pm 0.024$	15

In this table *l* and *r* stand for left and right, the side toward which the motion took place. The plus sign indicates that the observation was too late.

It will be noticed that the first and third observers have personal equations with minus values, that is, they anticipate the true time of the transit by an amount greater than their simple reaction-times. Compared with that standard, the second and fourth also anticipate, but not enough to make the values of their equations minus. The fifth waits till he has seen the star actually bisected or a little past bisection. But all are alike in observing at the equatorial and slower rates a little later with motion in the less accustomed direction, namely, toward the right; the anticipators anticipate less, and the tardy one is a little more behind. The differences, however, are not very certain till the slowest rates are reached.

The same tendency is shown in observations on real stars. Since 1884 a portion of the clock stars have been observed at Greenwich with motion from left to right by means of a prism eye-piece. From these and altazimuth observations the following comparative tables are composed:

PERSONAL EQUATION WITH MOTION *r* MINUS PERSONAL EQUATION WITH MOTION *l*.

Stars at a little less than equatorial rate.[1]

	Artificial stars.	Real stars.	No. of nights.
H T	$+0.027 \pm 0.020$	-0.035 ± 0.011	12
A D	$+0.014 \pm 0.012$	$+0.010 \pm 0.004$	70
T	$+0.042 \pm 0.010$	$+0.063 \pm 0.012$	28
L	$+0.017 \pm 0.011$	$+0.020 \pm 0.004$	72
H	$+0.011 \pm 0.011$	$+0.010 \pm 0.004$	85

Stars at rate of N. P. D. 43½°.

	Artificial stars.	Real stars (altazimuth).
H T	$+0.050 \pm 0.022$	$+0.041 \pm 0.020$
A D	$+0.020 \pm 0.011$	$+0.030 \pm 0.012$
T	$+0.046 \pm 0.010$	$+0.039 \pm 0.017$
L	$+0.011 \pm 0.010$	$+0.047 \pm 0.013$
H	$+0.014 \pm 0.011$	-0.015 ± 0.009

[1] The column marked "No. of nights" gives the number of occasions on which stars were observed with motion from left to right; it is the custom to observe two stars every fine night in this way.

The experimental results and those with real stars show but little variation.

C. Wolf explains the difference of personal equation in eye and ear observations as a result of asymmetry in the eye, and substantiates it in his own case by experiment. Such a peculiarity in structure is possibly not uncommon. With the chronograph method there is little chance for the operation of such a cause. It might, however, come in, for those that anticipate, in making their estimation of the distance of the star from the wire, but no explanation is yet put forward.

Personal equation is influenced by the rate of the star.—I have already spoken of the experiments of Bessel and C. Wolf in this particular (first part, pages 13 and 37). Bessel's results were negative; Wolf's are summarized in the following table:[1]

Rate.	P. E.	Error of P. E.
s.	s.	s.
31.5	+0.141	0.014
40.3	+0.120	0.010
53.1	+0.108	0.012
86.9	+0.091	0.015

The personal equation decreases as the length of time required to traverse the reticle increases, that is, as the rate of the star decreases; but the uncertainty is least at a rate somewhat greater than the equatorial, and increases with change in either direction.

The study of Wagner mentioned in the preceding section was designed chiefly to bring out the effect of very slow rates and the difference, under those circum-

[1] The rate is given by the number of seconds required to cross the reticle; the equatorial rate corresponded to a time of sixty seconds. The four rates are averages of groups; not the identical rates at which observations were made. It should be remembered also that all of Wolf's experiments were made by eye and ear.

stances, of personal equation by the two methods of observation. The motion of the artificial star was from left to right in the first set of experiments, from right to left in the second. The following tables are abstracted from those that summarize his results;[1] the rates are indicated by the declinations to which they corresponded:

FIRST SERIES, 1870.

Declination.	Absolute P. E. by eye and ear.	No. of series.	Absolute P. E. by chronograph.	No. of series.	Eye and ear minus chronographic.
°	s.		s.		s.
17.0	+ 0.010	3	+ 0.090	3½	— 0.080
40.1	— 0.009	5½	+ 0.061	7½	— 0.070
69.4	— 0.025	8	+ 0.070	7⅓	— 0.095
77.9	+ 0.086	6	+ 0.173	7	— 0.087
85 52′	+ 0.03	3	+ 0.14	3	— 0.11
86 27	+ 0.04	7	+ 0.31	7	— 0.27
88 22	— 0.20	7½	+ 0.32	7⅛	— 0.52
89 1	— 0.37	7	+ 0.17	7	— 0.54

SECOND SERIES, 1871.

Declination.	Absolute P. E. by eye and ear.	No. of series.	Absolute P. E. by chronograph.	No. of series.	Eye and ear minus chronographic.
30.4	+ 0.057	7	+ 0.065	7½	— 0.008
66.9	+ 0.029	8	+ 0.085	9	— 0.056
77.6	+ 0.002	8	+ 0.111	8	— 0.109
85 58′	— 0.06	7½	+ 0.29	7½	— 0.35
88 12	— 0.13	8	+ 0.33	8	— 0.46
89 1	— 0.40	8	+ 0.11	8	— 0.51

The personal equation with the chronographic method increases as the rate decreases, except for the fastest rate in the first set, where the trials were relatively few, and for the slowest rate of all. There is also an increasing tendency to anticipate in the eye and ear values of the second set, and the difference between the two methods is in both an increasing one. Results of the same general import as these by exper-

[1] In Wagner's tables the minus sign indicated an observation later than the phenomenon; the signs have here been changed throughout to facilitate comparison.

iment were obtained from observation of actual stars. It should not be forgotten, however, that most of these rates are extremely slow (the bisection here must have continued an appreciable time), and the results are therefore not strictly comparable with those for more rapid rates.

The latest experiments on this point, as on the last, are those at Greenwich, but they are not comparable with those of Wolf, because made by the chronographic method. The table on page 285 shows the effect of rate as well as of direction; there is little need of further comment upon them. For the first four observers the tendency to change is strongest when the motion is toward the left. The change is an increase in anticipation; the minus values become greater, the plus values less. With motion toward the right there is greater irregularity, but at the very slowest rate a tendency to delay. The last observer suffers an increase of personal equation with both directions of motion. Turner comments on the result of the study as follows: "There does not appear to be any general law representing the change of personal equation with rate. We might suppose the personal equation to be made up partly of error of bisection and partly of a constant error in time either deliberate or unconscious. We should expect the latter not to vary with the rate or direction of motion, and the former to vary inversely as the rate, and to change sign with the direction of motion. This is fulfilled approximately for all the observers except H, whose 'error of bisection,' amounting to nearly 1″.0, does not change sign with the direction of motion, so that in this case the observer apparently waits until the star has traveled about 1″ past the wire—whatever be the

direction of motion—and then 'hangs fire' for about 0.2 s before the record is complete. Such a well-marked exception rather precludes generalization."

Without attempting to give an explanation where Turner has declined, it may not be amiss to notice a single suggestive point. Gould[1] has remarked on the difficulty that those who anticipate must have in adjusting the amount of their anticipation to the changing rates of the stars. If they record for a slow moving star when it is as far from bisection as a rapid one would have to be, the amount of their anticipation expressed in time is increased. Something of the same sort, but of a contrary effect, would trouble those that allow the star to pass beyond the wire before recording. A slow-moving star would take longer in reaching the point at which they begin to record than a rapid one. Four of these observers anticipate, as we have said, and one delays beyond the bisection. Correspondingly, in the table we find, at least for the direction of most customary motion, an increase of personal equation with decreasing rates.

So the question stands. The fact that personal equation is affected by the rate is established, but not yet the law nor the manner of its influence.

The personal equation is affected by yet other circumstances of observation.—The preceding sections have been concerned with the influence of the character of the object; in this, I shall speak of several other factors—the position of the observer, the illumination of the reticle, the kind of key used, and the like.

In actual observation, the position of the observer changes with the declination, and so with the rate, of the star. It has therefore been felt by astronomers

[1] Cited by Gill from Gould's Transatlantic Longitudes.

that experiments on the effect of rate made, as in most cases, without change of position, did not fulfill the conditions. The point has, however, received some attention from the experimenters. C. Wolf[1] adapted his machine to observing when flat on the back, with the following result (series one and four were made in the normal position, for the sake of comparison; all were made by eye and ear): 0.113 s, 0.114 s, 0.125 s, 0.103 s. He thinks, however, that the difference does not prove a change depending on position, probably because his results were exact only to 0.01 s. Rogers[2] found that a very constrained position made his observations (chronographic) a little earlier (0.011 s in the mean) and, strange to say, somewhat more regular.

In 1879, Bakhuyzen suggested an apparatus for the observation of artificial transits in the positions of natural ones,[3] and in the past year Wislicenus, of the Observatory of Strasburg, has reported such observations made with an ingenious device of his own.[4] His absolute personal equation in the five positions is as follows:

Rate expressed as declination.	Zenith.	45°	Horizon.	−45°	Nadir.
	s.	s.	s.	s.	s.
11½°	−0.19	−0.18	−0.21	−0.13	+0.14
60	−0.01	−0.02	−0.05	+0.04	+0.50
80	+0.52	+0.48	+0.32	+0.41	+1.77

The amount, and especially the direction of the illumination of the field and the reticle have been found

[1] Loc. cit.

[2] Loc. cit.

[3] Vierteljahrssch. d. Astr. Gesell., XIV, 414.

[4] Noted in *The Observatory*, December, 1888, p. 443, where the following criticism is made: "It must be remembered, however, that in different positions of the telescope, the contacts of the recording apparatus [which moves with the telescope] might be sensibly different. Very little information is given as to the adjustment of this apparatus."

important for personal equation. C. Wolf thought the illumination of the field of little effect, but Newcomb enumerates the brighter field as one among other reasons that might co-operate to make day observations systematically earlier than those at night. Rogers found that he observed a few hundredths of a second later when the wires were faint than when they were bright.

R. Wolf, of Zürich, with his assistant, undertook a careful investigation of the effect of direction of illumination.[1] He was led to this by finding his relative personal equation with Hirsch to be of different sign and amount when determined at Zürich and at Neuchâtel. It was discovered that the direction was without influence so long as the position of the ocular exactly suited the eye of the observer. But if the ocular were drawn out or pushed in too far, the sign of the personal error changed with the side of the illumination, illumination of the field and of the reticle having contrary effects, and the amount (which could reach several tenths of a second) varied with the amount of displacement of the ocular. These differences were not found with double-sided nor with day illumination. In circumstances described in a later communication,[2] two images of the reticle could be seen, which changed their relative positions right and left as the direction of illumination changed and as the ocular was displaced inward or outward. These briefly mentioned papers, though having to do with instrumental conditions, are extremely important for all determinations of relative personal equation where the sources of error they point out have not been excluded. Per-

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[1] *Astronomische Nachrichten*, LXXV, 71.

[2] *Astronomische Nachrichten*, LXXVI, 369.

haps in some such way as this the enormous difference between Bessel and Argelander, and between some other pairs of astronomers, and likewise the progressive change of relative personal equation, are to be in part accounted for.

Personal equation is somewhat affected by the kind of key used in recording. Schiff reacted to an electrical stimulus on the tip of the tongue by means of a telegraph key, an "astronomical key," and a "book key," with these results: telegraph key, 0.156 s $\pm$ 0.023; "astronomical key," 0.164 s $\pm$ 0.016; "book key," 0.130 s $\pm$ 0.010. Tacchini found figures from observations on a Wolf artificial transit apparatus as follows: "astronomical key," 0.172 s $\pm$ 0.027; "book key," 0.166 s $\pm$ 0.016.[1] The key used in the Naval Observatory at Washington is something like a telegraph key, but of course held in the hand in observing. A small but genuine difference of personal equation is introduced if observers hold the finger at different heights above the knob when ready to make a record.

The study of Rogers included many circumstances besides those mentioned. Some of his results may be briefly mentioned here. A change of more than forty degrees in temperature, carrying the mercury below zero, produced no definite effect on his own observations, though a single set on each of two other observers showed a delay of 0.073 s and 0.107 s respectively. Fatigue had little effect, but records taken after sleep following fatigue were somewhat quicker. Sets taken at intervals during a fast of thirty hours were, on the

[1] Tacchini: Sulla equazione personale. *Rivista sicula di scienze, lettere ed arti.* Anno I, Vol. 2°, p. 382, cited by Buccola, La legge del tempo, p. 170. The "astronomical key" was a button held between the first two fingers and pressed with the thumb; the "box or book key" (*tasto a scatola od a libro*) of Secchi is pressed with all four fingers.

average, 0.032 s quicker than a normal set taken just before. The relative personal equation between Rogers and one of the other observers for a set of stars separated by from ten to fifteen seconds was 0.069 s greater than for a set separated by only two or three seconds. The relative personal equation for the same two observers was 0.023 s smaller for artificial than for natural stars.

The personal equation is influenced by circumstances more or less purely psychic.—The time of the perception of one sensation is affected by the regular recurrence of a second. And this is true not only in the eye and ear observations, where attention must be given to two, but also in the chronographic, where the second is merely accidental. H. Leitzmann discovered such an effect in chronographic records made within hearing of a loud ticking clock.[1] The seconds given by the clock were alternately too long and too short, the long being 1.05 s, the short 0.95 s. The tick that marked the beginning of each was also followed at about 0.35 s by another made with the forward motion of the second hand. Beginning with the first tick of the long second, the times were as follows: 0.00 s, 0.35 s, 1.05 s, 1.40 s.

Leitzmann was an anticipator, trying, as he says, "to make the feeling of movement that corresponds to the closing of the electric circuit in the key contemporaneous with the perception of the coincidence of the star and wire." In observing, he was accustomed, in addition to taking the transits of the vertical wires, to take also the star's declination by tapping his key when the star was midway between a pair of hori-

[1] Wundt's *Philos. Studien*, Bd. V, H. 1, S. 56.

zontal wires. In working up his results after the observations were complete, he found that those for declination grouped themselves on certain tenths of the second. In 129 observations the distribution was as follows :

Tenth of the second,	0	1	2	3	4	5	6	7	8	9
Distribution of obs.,	2	4	3	30	60	18	4	0	4	4

The last two-thirds of the time covered eighty-six observations ; seventy-six of them fell on 3 and 4, nine on 5, and one on 1. A connection with the clock beat was evident, and the observer went on to examine the more accurate transit observations for traces of a similar disturbance. Such he was able to find.

Assuming that the differences that remained after reducing the observations at the individual wires to the central one were due chiefly to the disturbance in question, he distributed the amounts so found in groups corresponding to each fiftieth of a second, averaged them, and plotted curves from the averages. In his curves, plus ordinates correspond to an increase of personal equation; minus, to a decrease. The curves are somewhat irregular, but a typical one would show a short wave followed by a longer one, in each of which there is first a descending and then an ascending phase. The first wave would cover the first four tenths of the second, the other the remainder. The points of greatest personal equation fall near the beginning and end of the curve, those of least in the middle of the second portion. The variations that the curves represent amount at the greatest to about 0.054 s, and the mean probable error of each value to about 0.004 s. Leitzmann attributes the changes to the influence of the intruding stimulus on attention, which influence was undoubtedly strengthened in his own case by a

habit of semi-consciously counting the beats as in observing by the other method. Into the details of his exposition, some of which seem to me open to criticism, I shall not enter here.

In observations by eye and ear, many observers show a preference for some special tenth of the second, so that it recurs in their recorded observations far more frequently than it should by chance, and this "mental time scale" is often unchanged by time or circumstance.[1] Whether this is to be attributed to slip-shod habits of estimation or to some untraceable association like those that give numbers and letters peculiar characters in the minds of some people, or to some effect of the clock-beat like that just described, it is impossible to say without fuller information. Similar, but involving retinal asymmetry, is the personal difference that affects the reading of chronograph records from the record sheet, that is, the fixing the relative distance of the jag in the line that marks the transit from the other jags that mark the seconds.

Surprise is a psychological factor that is hardly to be expected in transit observations where the observer sees the star advancing in the field and knows precisely what he is to do when it reaches the reticle. It seems to be a fact, however, that records at the first wire are a little less certain than at the rest.[2] The transits of the wires after the first recur at intervals with which an observer in time becomes so familiar that he could even make some sort of a record at the remaining wires for a star that disappeared after

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[1] B. Peirce: Proc. American Acad. of Arts and Sciences, IV (1860), 197.

[2] Argelander: *Vierteljahrss. d. Astron. Gesells.*, 1872, p. 16, referred to by Downing (*Monthly Notices*, XXXVIII), who adds figures for probable error at first and ninth wire in observations of the sun.

crossing the first. The knowledge of the exact time at which the transit is going to happen enables him to make full mental and physical preparation for its observation.

Knowledge of the amount and direction of the personal error may lead to change in it. C. Wolf was able by three months' practice to reduce his anticipation in eye and ear observations permanently from 0.30 s to 0.11 s. Rogers found the assumption that he observed too late sufficient to quicken his observations by more than 0.03 s. Wolf probably had, like Rogers, the expectation of realizing such a change under such circumstances, and that expectation may have co-operated in producing the result.

The foregoing exposition of circumstances under which the personal equation is variable probably does not exhaust the catalogue. Everything, physical or psychical, that influences the nervous mechanism influences in some degree the time of perceiving and recording the perception.[1] Psychological experimenters have found measurable differences in reaction times depending on attention, practice, fatigue, intensity of stimulus, effect of drugs, emotional states and disease, and other differences that some, at least, have thought due to grade of education, race, age, sex, quality of stimulus, and temperature.

I may sum up the general result of the preceding pages as follows: The personal equation by both methods varies from assignable and unassignable causes. Among the former are the nature of the object (disk or star), its magnitude, its direction and

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[1] How responsive that mechanism is appears in such experiments as those of Lombard on the Knee-jerk, AMERICAN JOURNAL OF PSYCHOLOGY, Vol. I, p. 5.

rate of motion, and sundry instrumental and psychic conditions. The explanations of the changes, more or less conjecturally advanced, point to individual differences in the effect of irradiation, in the structure or action of the eye, in habits of anticipation or delay in observing, in the form of recording (Wundt), and in the direction of attention. It is impossible to establish by the observations and experiments described the certain causes of the differences found. The investigations of the astronomers have generally had the very practical aim of discovering the corrections to be applied to their observations. They have therefore tried to reproduce as nearly as possible the complicated conditions of their regular observations. It is for this reason that such a review as has been made is much more fruitful in problems for physiology and psychology than in generalizations. For the purposes of these two sciences the experimental conditions must be simplified and varied, not complicated and held to a typical form.

On the general question of the nature of personal equation itself apart from its variations, the experiments have been a little more explicit and the work of the astronomers and psychologists has come nearer together. It is hoped to treat of the nature and cause of personal equation in the concluding article of this series.